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Unsteady heat transfer to pulsatile flow of a dusty viscous incompressible fluid in a channel

N. DATTA and D. C. DALAL
Dept. of Mathematics, Indian Institute of Technology, Kharagpur-721 302, India

and

S. K. MISHRA
Balimela College of Science & Technology, Balimela-764 051, Orissa, India

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Abstract—The problem of unsteady heat transfer to pulsatile flow of a dusty fluid in a parallel plate channel has been studied. It is observed that the unsteady part of the fluid velocity as well as of the particle phase velocity has a phase lag which increases with increase of ϕ, i.e. the volume fraction of the particles. The steady part of the heat transfer at the hotter plate decreases with increase of ϕ whereas it increases with increase of ϕ at the colder plate. The amplitude of the unsteady part of the heat transfer at both the plates decreases with increase of ϕ.

INTRODUCTION

THE STUDY of heat transfer to a dusty fluid flowing in a channel has applications in technological fields, e.g. heat exchanger, reactor cooling etc. Further, considering blood as a binary system of plasma (fluid phase) and blood cells (particle phase), the study of dusty fluid and heat transfer has a relevance to the flow of blood.

In most of the studies of dusty fluid flows, the volume fraction of the particles has been neglected. However, this assumption is not justified when the fluid density is high or particle mass fraction is large. Rudinger [1] has shown that the error in neglecting the volume fraction range from insignificant to large. Nag and Datta [2, 3] have considered the volume fraction in the unsteady flow of a dusty fluid through a rectangular channel. Datta and Das [4] studied heat transfer in the flow of a dusty gas.

In the present study we have considered the problem of unsteady heat transfer to pulsatile flow of a dusty fluid in a parallel plate channel.

MATHEMATICAL FORMULATION

We consider the pulsatile flow of a dusty fluid between two infinitely long parallel plates at a distance h apart. Taking x-axis along the plates and y-axis normal to them, the pulsatile flow is assumed to be induced by the pressure gradient of the form

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = A[1+\varepsilon\, e^{i\omega t}], \tag{1}$$

A being a constant and $i = \sqrt{(-1)}$.

Since the plates are infinite, all physical quantities excepting pressure may be taken as functions of y and t only.

The governing equations of motion and energy for the two phases may be written as [5]:

$$(1-\phi)\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \rho_p(u_p-u)/(\rho\tau_p) \tag{2}$$

$$(1-\phi)\rho c_p\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \rho_p(u_p-u)^2/\tau_p + \rho_p c_s(T_p-T)/\tau_T \tag{3}$$

$$\frac{\partial u_p}{\partial t} = -\nu_p\frac{\partial^2 u_p}{\partial y^2} - (u_p-u)/\tau_p \tag{4}$$

$$\rho_p c_s\frac{\partial T_p}{\partial t} = -\rho_p c_s(T_p-T)/\tau_T - \mu_p\left(\frac{\partial u_p}{\partial y}\right)^2 - \mu_p u_p\frac{\partial^2 u_p}{\partial y^2}. \tag{5}$$

Assuming the plates to be maintained at constant temperature, the boundary conditions of the problem are

$$\left.\begin{aligned} u=0,\quad u_p=0,\quad T=T_0 \quad &\text{at}\quad y=0,\\ u=0,\quad u_p=0,\quad T=T_1 \quad &\text{at}\quad y=h \end{aligned}\right\} \tag{6}$$

where it is assumed that $T_1 > T_0$.

METHOD OF SOLUTION

Introducing the following dimensionless variables and parameters,

NOMENCLATURE

c_p, c_s	specific heats of fluid and solid particles
Ec	Eckert number
f	ratio between ρ_p and ρ
h	distance between plates
k	thermal conductivity
p	pressure of fluid
Pr	Prandtl number
R, R_p	fluid and particle phase Reynolds numbers
t	time
T, T_p	temperatures of fluid and particle phase
T_0, T_1	temperatures of the plate at $\eta = 0$ and at $\eta = 1$
u, u_p	fluid and particle phase velocities along x-axis
u_0, u_{p0}	steady parts of u and u_p
u_1, u_{p1}	unsteady parts of u and u_p
x, y	space coordinates along and perpendicular to the plates.

Greek symbols

α	dust parameter
γ	ratio between c_s and c_p
θ, θ_p	dimensionless fluid and particle phase temperatures
θ_0, θ_{p0}	steady parts of θ and θ_p
θ_1, θ_2	unsteady parts of θ
θ_{p1}, θ_{p2}	unsteady parts of θ_p
μ	viscosity of fluid
ν, ν_p	kinematic viscosities of fluid and particle phase
ξ, η	dimensionless values of x and y
ρ, ρ_p	densities of fluid and particle phase
ρ_c	material density of solid particles
τ_p, τ_T	particle velocity and thermal relaxation times
ϕ	volume fraction of dust particles
ω	the frequency of oscillation.

$$
\begin{gathered}
\bar{u} = u\omega/A, \quad \theta = (T-T_0)/(T_1-T_0), \quad \bar{t} = t\omega, \\
\xi = x/h, \quad \bar{u}_p = u_p\omega/A, \quad \theta_p = (T_p-T_0)/(T_1-T_0), \\
\bar{p} = p/(A\rho h), \quad \eta = y/h, \quad R = \omega h^2/\nu, \\
R_p = \omega h^2/\nu_p, \quad f = \rho_p/\rho, \quad \gamma = c_s/c_p, \\
Pr = \mu c_p/k, \quad Ec = -A^2/\{\omega^2 c_p(T_1-T_0)\}
\end{gathered}
$$

the equations (1)–(4) can be rewritten as, on dropping the bars for convenience,

$$
-\frac{\partial p}{\partial \xi} = 1 + \varepsilon e^{it} \tag{7}
$$

$$
(1-\phi)\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial \xi} + \frac{1}{R}\frac{\partial^2 u}{\partial \eta^2} + \alpha f(u_p - u) \tag{8}
$$

$$
\frac{\partial u_p}{\partial t} = -\frac{1}{R_p}\frac{\partial^2 u_p}{\partial \eta^2} - \alpha(u_p - u) \tag{9}
$$

$$
\begin{aligned}
(1-\phi)\frac{\partial \theta}{\partial t} &= \frac{1}{R\,Pr}\frac{\partial^2 \theta}{\partial \eta^2} + \frac{Ec}{R}\left(\frac{\partial u}{\partial \eta}\right)^2 \\
&\quad + \alpha f Ec(u_p - u)^2 + \frac{2}{3}\frac{\alpha f \gamma}{Pr}(\theta_p - \theta)
\end{aligned} \tag{10}
$$

$$
\frac{\partial \theta_p}{\partial t} = -\frac{Ec}{\gamma R_p}\left(\frac{\partial u_p}{\partial \eta}\right)^2 - \frac{Ec}{\gamma R_p}u_p\frac{\partial^2 u_p}{\partial \eta^2} - \frac{2}{3}\frac{\alpha}{Pr}(\theta_p - \theta). \tag{11}
$$

Since the flow is induced by the pressure gradient of the form given in equation (7), the velocity and temperature of the fluid and that of the particle phase can be assumed as,

$$
\left.\begin{aligned}
u &= u_0(\eta) + \varepsilon u_1(\eta)\,e^{it} \\
u_p &= u_{p0}(\eta) + \varepsilon u_{p1}(\eta)\,e^{it}
\end{aligned}\right\} \tag{12}
$$

$$
\left.\begin{aligned}
\theta &= \theta_0(\eta) + \varepsilon\theta_1(\eta)\,e^{it} + \varepsilon^2\theta_2(\eta)\,e^{2it} \\
\theta_p &= \theta_{p0}(\eta) + \varepsilon\theta_{p1}(\eta)\,e^{it} + \varepsilon^2\theta_{p2}(\eta)\,e^{2it}
\end{aligned}\right\}. \tag{13}
$$

Using equations (7), (12) and (13) in equations (8)–(11) and comparing the terms free from ε, the terms with ε and ε^2 respectively, we get the following sets of equations

$$
\left.\begin{aligned}
&\frac{1}{R}\frac{d^2 u_0}{d\eta^2} + \alpha f(u_{p0} - u_0) = -1 \\
&\frac{1}{R_p}\frac{d^2 u_{p0}}{d\eta^2} + \alpha(u_{p0} - u_0) = 0 \\
&\frac{1}{R\,Pr}\frac{d^2\theta_0}{d\eta^2} + \frac{2}{3}\frac{\alpha f\gamma}{Pr}(\theta_{p0} - \theta_0) + \frac{Ec}{R}\left(\frac{du_0}{d\eta}\right)^2 \\
&\qquad + \alpha f Ec(u_{p0} - u_0)^2 = 0 \\
&\frac{2}{3}\frac{\alpha\gamma}{Pr}(\theta_{p0} - \theta_0) + \frac{Ec}{R_p}\left(\frac{du_{p0}}{d\eta}\right)^2 + \frac{Ec}{R_p}u_{p0}\frac{d^2 u_{p0}}{d\eta^2} \\
&\qquad = 0
\end{aligned}\right\} \tag{14}
$$

$$
\left.\begin{aligned}
&\frac{1}{R}\frac{d^2 u_1}{d\eta^2} + \alpha f(u_{p1} - u_1) - i(1-\phi)u_1 = -1 \\
&\frac{1}{R_p}\frac{d^2 u_{p1}}{d\eta^2} + \alpha(u_{p1} - u_1) + iu_{p1} = 0 \\
&\frac{1}{R\,Pr}\frac{d^2\theta_1}{d\eta^2} + \frac{2}{3}\frac{\alpha f\gamma}{Pr}(\theta_{p1} - \theta_1) + \frac{2Ec}{R}\frac{du_0}{d\eta}\frac{du_1}{d\eta} \\
&\quad + 2\alpha f Ec(u_{p0} - u_0)(u_{p1} - u_1) - i(1-\phi)\theta_1 = 0 \\
&\frac{2\alpha\gamma}{3Pr}(\theta_{p1} - \theta_1) + i\gamma\theta_{p1} + \frac{Ec}{R_p}\left(2\frac{du_{p0}}{d\eta}\frac{du_{p1}}{d\eta}\right. \\
&\qquad \left. + u_{p0}\frac{d^2 u_{p1}}{d\eta^2} + u_{p1}\frac{d^2 u_{p0}}{d\eta^2}\right) = 0
\end{aligned}\right\} \tag{15}
$$

$$\left.\begin{aligned}&\frac{1}{R\,Pr}\frac{d^2\theta_2}{d\eta^2}+\frac{2}{3}\frac{\alpha f\gamma}{Pr}(\theta_{p2}-\theta_2)+\frac{Ec}{R}\left(\frac{du_1}{d\eta}\right)^2\\&\qquad+\alpha f Ec(u_{p1}-u_1)^2-2i(1-\phi)\theta_2=0\\&\frac{2}{3}\frac{\alpha\gamma}{Pr}(\theta_{p2}-\theta_2)+\frac{Ec}{R_p}\left(\frac{du_{p1}}{d\eta}\right)^2+\frac{Ec}{R_p}u_{p1}\frac{d^2u_{p1}}{d\eta^2}\\&\qquad+2i\gamma\theta_{p2}=0\end{aligned}\right\}\quad(16)$$

The corresponding boundary conditions are,

$$\left.\begin{aligned}u_0=0,\quad u_{p0}=0,\quad \theta_0=0\quad\text{at}\quad \eta=0,\\ u_0=0,\quad u_{p0}=0,\quad \theta_0=1\quad\text{at}\quad \eta=1\end{aligned}\right\}\quad(17)$$

$$\left.\begin{aligned}u_1=0,\quad u_{p1}=0,\quad \theta_1=0\quad\text{at}\quad \eta=0,\\ u_1=0,\quad u_{p1}=0,\quad \theta_1=0\quad\text{at}\quad \eta=1\end{aligned}\right\}\quad(18)$$

$$\left.\begin{aligned}\theta_2=0\quad\text{at}\quad \eta=0,\\ \theta_2=0\quad\text{at}\quad \eta=1\end{aligned}\right\}.\quad(19)$$

From the first two equations of (14) eliminating u_p we get,

$$\frac{d^4u_0}{d\eta^4}+m_0^2\frac{d^2u_0}{d\eta^2}+m_1=0\quad(20)$$

where $m_0^2=R_p\alpha-R\alpha f$ and $m_1=RR_p\alpha$.

Our objective is to discuss that type of flow in which the volume fraction ϕ of the particle phase is small but not negligible and particles will diffuse throughout the carrier fluid. So we can assume that the viscosity of particle phase is very small compared to that of the carrier fluid (i.e. $v_p \ll v$), so that $R_p > R$ and $f<1$ which make m_0^2 always positive.

Solving the equation (20) for u_0 using boundary conditions from (17) we have,

$$u_0=\frac{m_1}{2m_0^2}\eta(1-\eta)+\frac{Rm_0^2-m_1}{m_0^4}\left\{\frac{\cos(m_0\eta-m_0/2)}{\cos(m_0/2)}-1\right\}\quad(21)$$

using expression of u_0 in the first equation of (14), u_{p0} can be written as,

$$u_{p0}=\frac{m_1}{2m_0^2}\eta(1-\eta)-\frac{m_1}{m_0^4}\left\{\frac{\cos(m_0\eta-m_0/2)}{\cos(m_0/2)}-1\right\}.\quad(22)$$

Similarly, eliminating u_{p1} from the first two equations of (15) and solving the resulting differential equation for u_1 with appropriate boundary conditions from (18) we get,

$$u_1=A_1\frac{e^{\alpha_1\eta}+e^{\alpha_1(1-\eta)}}{1+e^{\alpha_1}}+B_1\frac{e^{\beta_1\eta}+e^{\beta_1(1-\eta)}}{1+e^{\beta_1}}+k_4\quad(23)$$

where,

$$A_1=\frac{k_4\beta_1^2-R}{\alpha_1^2-\beta_1^2},\qquad B_1=\frac{R-k_4\alpha_1^2}{\alpha_1^2-\beta_1^2},$$

$$\alpha_1^2=\frac{-k_1+\sqrt{(k_1^2-4k_2)}}{2},\quad \beta_1^2=\frac{-k_1-\sqrt{(k_1^2-4k_2)}}{2}.$$

$$k_4=\frac{k_3}{k_2},\qquad k_2=RR_p(1-\phi)-iRR_p\alpha\{f+(1-\phi)\},$$

$$k_3=-RR_p(\alpha+i),$$

$$k_1=\alpha(R_p-Rf)+i\{R_p-R(1-\phi)\}.$$

Using u_1 in the first equation of (15) the expression for u_{p1} can be obtained as,

$$u_{p1}=\left[1+\frac{i(1-\phi)}{\alpha f}\right]u_1-\frac{1}{\alpha f}-\frac{1}{R\alpha f}\times\left[A_1\alpha_1^2\frac{e^{\alpha_1\eta}+e^{\alpha_1(1-\eta)}}{1+e^{\alpha_1}}+B_1\beta_1^2\frac{e^{\beta_1\eta}+e^{\beta_1(1-\eta)}}{1+e^{\beta_1}}\right].\quad(24)$$

Eliminating θ_{p0} from the last two equations of (14) and using the solutions of u_0 and u_{p0} in the resulting differential equation of θ_0, the solution has been obtained on applying the boundary conditions for θ_0 from (19),

$$\begin{aligned}\theta_0={}&A_2\eta-B_2\eta^2+C_2(2-\eta)\eta^3\\&+D_2\left[\frac{(2Rm_0^2+1)\cos(2m_0\eta-m_0)}{8\cos(m_0/2)}\right.\\&\left.-(2Rm_0^2+3m_1+2)\cos\left(m_0\eta-\frac{m_0}{2}\right)\right]\\&+E_2\left[\frac{(1-2\eta)}{m_0^3}\sin\left(m_0\eta-\frac{m_0}{2}\right)\right.\\&\left.-\frac{(1-\eta)\eta}{2m_0^2}\cos\left(m_0\eta-\frac{m_0}{2}\right)\right]+F_2\end{aligned}\quad(25)$$

where

$$A_2=\frac{Ec\,Pr\,R}{2m_0^2}\left[\frac{m_1}{4}\left(1-\frac{2\alpha R_p}{3}+R\alpha f\right)+\frac{f\alpha}{m_0^4}\left(1+R^2m_0^2+\frac{R}{2\cos^2}\left(\frac{m_0}{2}\right)\right)\right]+1$$

$$B_2=\frac{R\,Ec\,Pr}{8m_0^4}\left[m_1m_0^2+\frac{4\alpha f(1+R^2m_0^2)}{m_0^2}+\frac{2R\alpha f}{m_0^2\cos^2(m_0/2)}\right]$$

$$C_2=\frac{Ec\,Pr}{24R_p}\left(\frac{m_1}{m_0}\right)^2(2R_p-3Rf)$$

$$D_2=\frac{Ec\,Pr\,R^2\alpha f}{m_0^2\cos(m_0/2)}$$

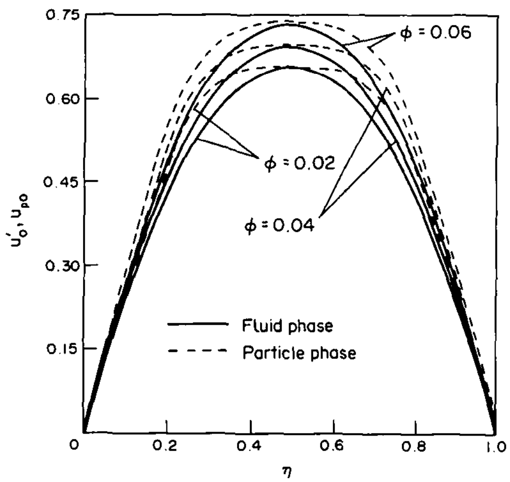

FIG. 1. Distribution of u_0 and u_{p0}.

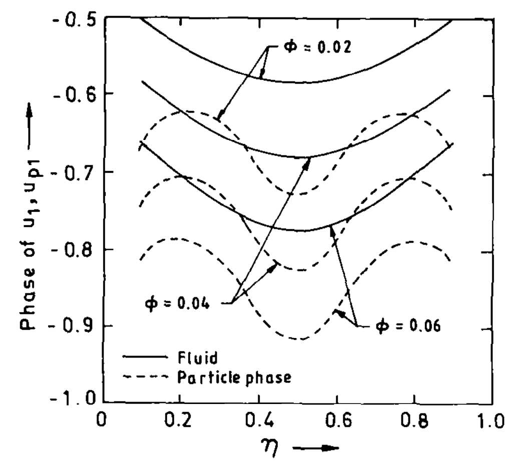

FIG. 3. Distribution of phase lag of u_1 and u_{p1}.

$$E_2 = \frac{Ec\,Pr\,Rf}{R_p \cos(m_0/2)}(m_1/m_0^2)^2$$

$$F_2 = D_2\left[2Rm_0^2 + 3m_1 - \frac{1-2Rm_0^2\cos(m_0)}{8\cos(m_0/2)} + m_0 m_1 \tan\left(\frac{m_0}{2}\right)\right]\cos\left(\frac{m_0}{2}\right)$$

From the last equation of (14) using expressions of u_0 and u_{p0} from (21) and (22), respectively, we get,

$$\theta_{p0} = \theta_0 - \frac{3}{2}\frac{Pr\,Ec}{\gamma\alpha R_p m_0^4} \times \left[\left(R_p R^2 - \frac{m_1^2}{m_0^2}\right)\left\{\frac{\cos(m_0\eta - m_0/2)}{\cos(m_0/2)} - 1\right\}^2 + \frac{m_1^2}{4}\left\{1 - 2\eta + \frac{2}{m_0}\frac{\sin(m_0\eta - m_0/2)}{\cos(m_0/2)}\right\}^2 + \frac{m_1^2}{2}\eta(1-\eta)\left\{\frac{\cos(m_0\eta - m_0/2)}{\cos(m_0/2)} - 1\right\}\right]. \quad (26)$$

From the last two equations of (15), θ_{p1} has been eliminated and the resulting differential equation of θ_1 has been solved using appropriate boundary conditions from (18). Using the expression for θ_1 in the last equation of (15), the expression for θ_{p1} can be calculated. Applying the same procedure in the two equations of (16) and using boundary conditions from (19) the solution of θ_2 and θ_{p2} can easily be obtained. To save space, the expressions for θ_1, θ_{p1}, θ_2 and θ_{p2} have been omitted.

DISCUSSION

In order to make a detailed discussion of results, numerical computations have been made on taking $R = 5.0$, $R_p = 20.0$, $\alpha = 10$, $Pr = 0.72$, $Ec = 0.02$, $\gamma = 1.4$, $\phi = (0.01\text{–}0.1)$ and $f = (\rho_c/\rho)\phi$, where we have assumed $\rho_c/\rho \approx O(10)$ and the results have been shown in Figs. 1–7.

Figure 1 shows that the distribution of the steady part of the velocity of fluid as well as that of the

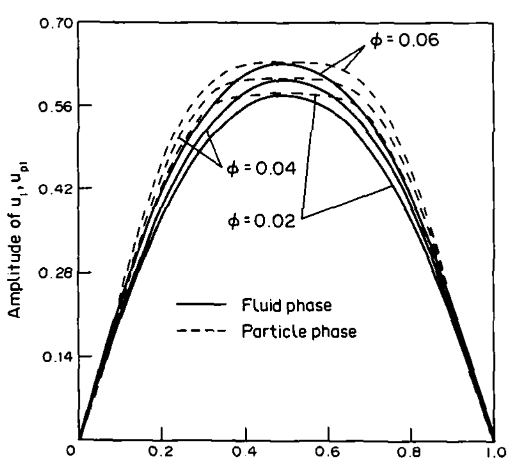

FIG. 2. Distribution of the amplitude of u_1 and u_{p1}.

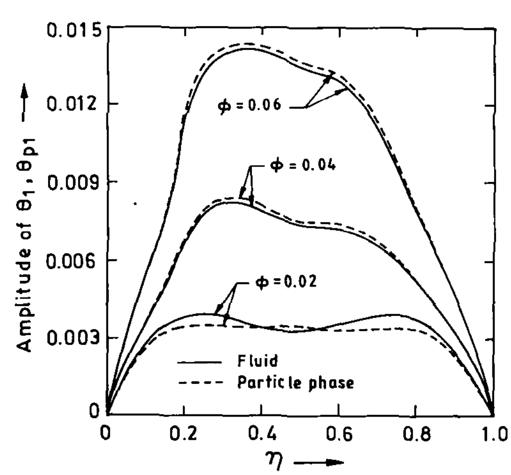

FIG. 4. Distribution of amplitude of θ_1.

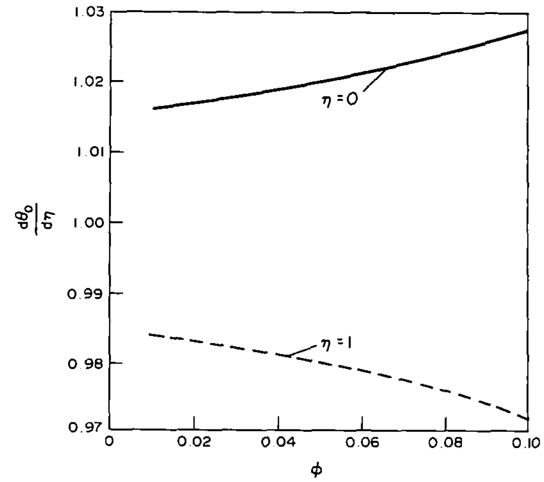

FIG. 5. Steady part $d\theta_0/d\eta$ of heat transfer of fluid phase against ϕ at $\eta = 0$ and $\eta = 1$.

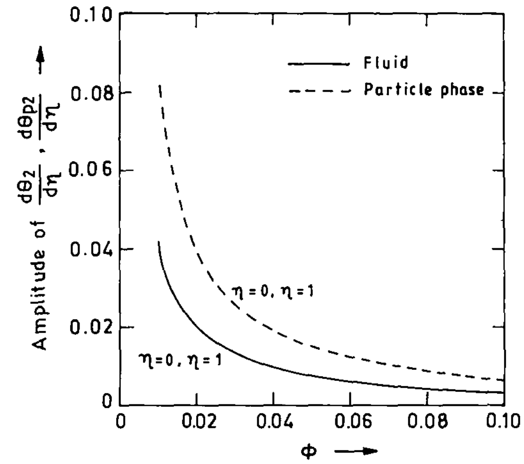

FIG. 7. Amplitude of unsteady part $d\theta_2/d\eta$ of heat transfer of fluid phase against ϕ at $\eta = 0$ and $\eta = 1$.

particle phase for various values of volume fraction ϕ. It reveals that for both phases the velocity increases with increase of ϕ and it may be noted that unlike the profiles of the fluid velocity, the profiles of the particle phase velocity are flatter at the centre of the channel.

In Fig. 2 the graphs of the amplitude of the unsteady part of the velocity of the fluid and that of the particle phase have been drawn against η for different values of ϕ. It can be observed that the amplitude increases with increase of ϕ for both phases. The profiles of the particle phase show that the amplitude remains the same for some distance near the centre of the channel.

Figure 3 shows the variation of phase lag of the unsteady part of the velocity of fluid as well as of particle phase against η for various values of ϕ. It shows that phase lag increases with increase of ϕ and at the centre of the channel the phase lag is maximum for both phases.

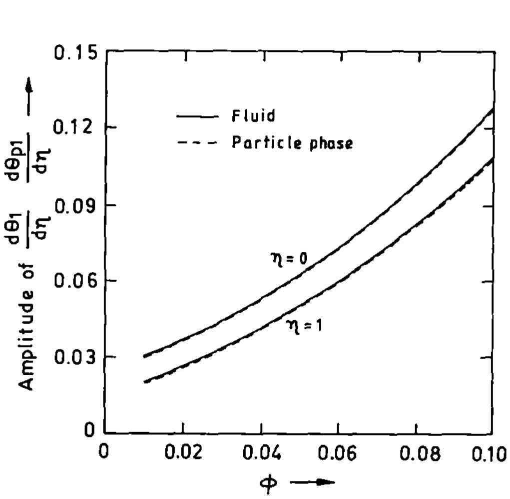

FIG. 6. Amplitude of unsteady part $d\theta_1/d\eta$ of heat transfer of fluid phase against ϕ at $\eta = 0$ and $\eta = 1$.

Figure 4 shows the variation of the amplitude of first unsteady part of the temperature of the fluid and that of the particle phase against η for various ϕ. It is observed that amplitude increases with increase of ϕ and reaches maximum near the colder wall.

The graphs of the steady part of heat transfer $d\theta_0/d\eta$ at $\eta = 0$ and $\eta = 1$ against ϕ have been drawn in Fig. 5. It shows that at the colder plate $\eta = 0$, $d\theta_0/d\eta$ increases with increase of ϕ and at the hotter plate $\eta = 1$, $d\theta_0/d\eta$ decreases with increase of ϕ. Heat transfer rate at $\eta = 1$ is always less than that at $\eta = 0$.

Figure 6 shows the graphs of amplitude of unsteady part of heat transfer, $d\theta_1/d\eta$ and $d\theta_{p1}/d\eta$ against ϕ for $\eta = 0$ and $\eta = 1$. It reveals that the amplitude of $d\theta_1/d\eta$ increase with increase of ϕ and the values of amplitude of $d\theta_1/d\eta$ and $d\theta_{p1}/d\eta$ at $\eta = 0$ are greater than those at $\eta = 1$.

Figure 7 shows the graphs of amplitude of unsteady part of heat transfer $d\theta_2/d\eta$ and $d\theta_{p1}/d\eta$ against ϕ for $\eta = 0$ and $\eta = 1$. It reveals that for small ϕ, the amplitude of $d\theta_2/d\eta$ and $d\theta_{p1}/d\eta$ increase with increase of ϕ.

CONCLUSION

This study reveals that the unsteady pulsatile flow of a dusty fluid has a phase lag which increases with increase of particle loading. The amplitude of the unsteady part of the heat transfer increases with increase of particle loading.

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